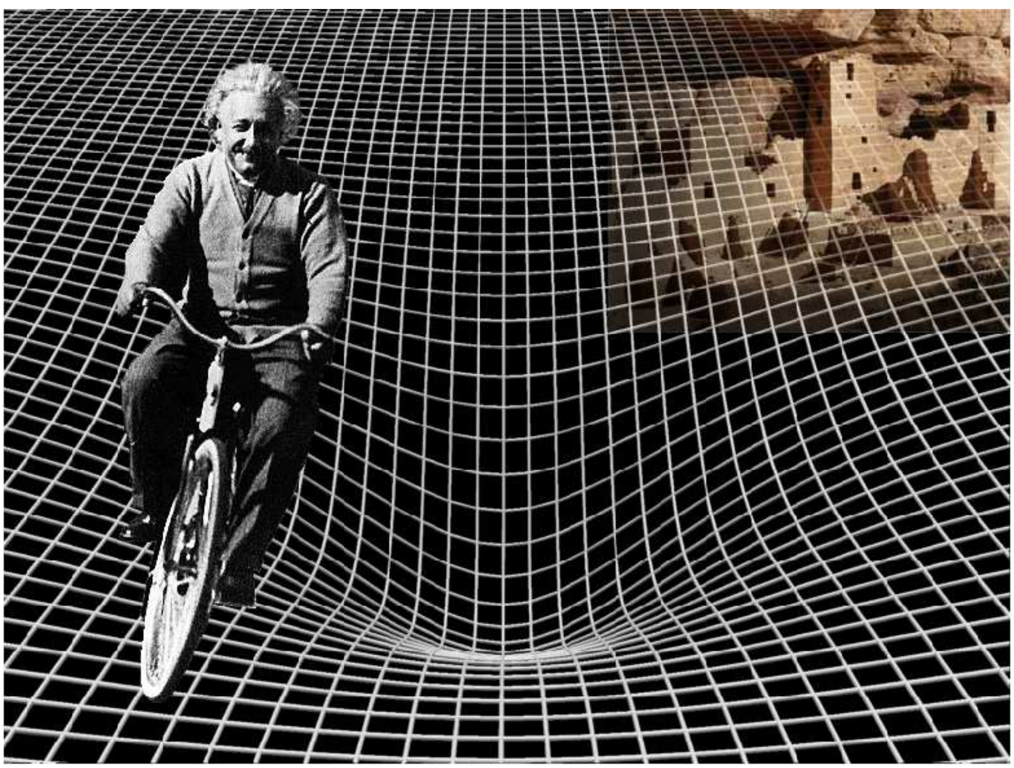
Formation & Growth of SMBHs: Simulations in General Relativity

Stuart L. Shapiro
University of Illinois at Urbana-Champaign



Physics and Astrophysics of Supermassive Black Holes

Santa Fe, New Mexico

July 9–14, 2006

Introduction and Motivation

- Compelling evidence:
  - SMBHs with $M \sim 10^6 - 10^{10} M_\odot$ are the engines that power quasars and AGNs.
  - SMBHs reside in most, & perhaps all, bulge galaxies, including the Milky Way.
- Still unknown:
  cosmological origin of seed SMBHs:
  - hydrodynamical stellar collapse?
  - collisionless matter collapse?
  - SIDM halo collapse?
  - massive scalar field or GW collapse?
- Strategy:
  BHs are strong-field objects governed by Einstein's theory of general relativity.
  $\Rightarrow$ GR simulations of
  - collapse to BHs,
  - BH binary merger and recoil,
  - BH accretion, etc.,

  may help reveal how, when and where SMBH seeds form and grow.

Clues and Constraints

- 1^{st} SMBHs:
  Existence of QSO SDSS 1148+5251 at $z_{QSO} = 6.43$ (Fan et al. 2003) $\Rightarrow$ 1^{st} SMBHs formed by $t = 0.87$ Gyr in ΛCDM model.

- Broad-line quasars with $0.1 \leq z \leq 2.1$:
  SDSS sample of 12,698 quasars obeys the Edd limit, $L_{bol} \lesssim L_{\mathrm{E}}$. (McLure & Dunlop 2004)

- Radiation efficiency:
  The luminosity density of quasars is $\sim 10\%$ the local SMBH mass density.
  (Soltan 1982; Yu & Tremaine 2002; Elvis et al. 2002)

$\Rightarrow$ An appreciable fraction of the mass of a SMBH is likely acquired by (baryonic) disk accretion.

$\Rightarrow$ The more massive the initial seed, the less time is required for it to grow to SMBH size by $z_{QSO} \geq 6.43$.

Stellar Progenitors of SMBH Seeds

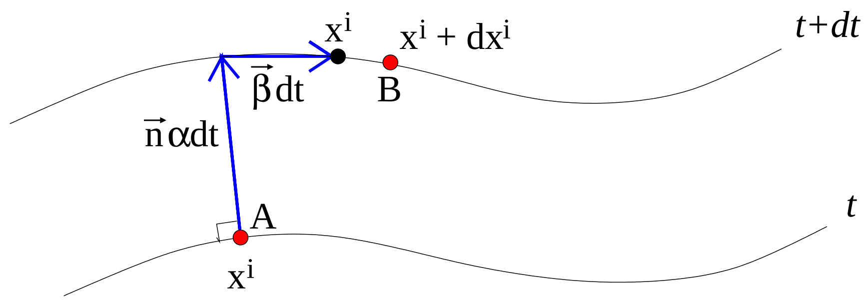
- One Possibility: a SMS, $M \gtrsim 10^4 M_\odot$.
  Form when contracting gas builds up sufficient rad'n pressure to inhibit fragmentation & prevent star formation.
  (e.g., Gnedin 2001; Bromm & Loeb 2003)

- GR rotating collapse simulations:
  max rotation yields a SMBH + disk,
  $M_h/M \approx 0.9,\ a_h/M_h \approx 0.75,\ M_D/M \approx 0.1.$
  (Shibata & Shapiro 2002)

- Problems:
  - SMSs have never been observed.
  - Simulations $\Rightarrow$ 1^{st} generation stars are
  Pop III stars, $M \approx 10^2 - 10^3 M_\odot$, not SMSs.
  (Bromm et al. 2002; Abel et al. 2002; Yoshida et al. 2006)

- Most conservative hypothesis:
  Pop III stars $\rightarrow$ BH seeds (Madau & Rees 2001):
  $M \sim 60 - 140$, & $\gtrsim 240 M_\odot$ (Heger et al. 2003);
  $M \lesssim 600 M_\odot$ (Onukai & Palla 2003; Yoshida et al. 2003)

3+1 (ADM) Field Eqns



$$ds^2 = -\underbrace{\alpha^2}_{\text{lapse}} dt^2 + \underbrace{\gamma_{ij}}_{\text{3-metric}} (dx^i + \beta^i dt)(dx^j + \underbrace{\beta^j}_{\text{shift}} dt) \ .$$

- Constraint Equations

$$\begin{aligned} R + K^2 - K_{ij}K^{ij} &= 16\pi\rho \quad \text{(Hamiltonian)} \ , \\ D_j(K^{ij} - \gamma^{ij}K) &= 8\pi S^i \quad \text{(Momentum)} \ . \end{aligned}$$

- Evolution Equations

$$\begin{aligned} \partial_t\gamma_{ij} &= -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i \ , \\ \partial_t K_{ij} &= \alpha R_{ij} + \cdots - 8\pi\alpha[S_{ij} - \frac{1}{2}\gamma_{ij}(S-\rho)] \ . \end{aligned}$$

- Gauge Quantities

$$\alpha, \quad \beta^i$$

Modified ADM Field Eqns

Shibata & Nakamura 1995; Baumgarte & Shapiro 1999

(BSSN)

- Conformal Decomposition: "York-Lichnerowicz split"

$$\tilde{\gamma}_{ij} = e^{-4\phi}\gamma_{ij}, \quad \text{where} \quad e^{4\phi} = \gamma^{1/3} ,$$
$$\tilde{A}_{ij} = \tilde{K}_{ij} - \frac{1}{3}\tilde{\gamma}_{ij}K$$

- Connection Functions

$$\tilde{\Gamma}^i \equiv \tilde{\gamma}^{jk}\tilde{\Gamma}^i{}_{jk} = -\partial_j\tilde{\gamma}^{ij} ,$$

- Evolve

$$\tilde{\gamma}_{ij}, \quad \tilde{A}_{ij}, \quad \phi, \quad K, \quad \& \quad \tilde{\Gamma}^i$$

- Advantage

$$\tilde{R}_{ij} = -\frac{1}{2}\underbrace{\tilde{\gamma}^{lm}\partial_m\partial_l\tilde{\gamma}_{ij}}_{\text{`}Laplacian\text{`}} + \underbrace{\tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k}_{\text{remaining 2nd derivs}} + \cdots ,$$

$$\Rightarrow \quad \partial_t^2\tilde{\gamma}_{ij} \sim \partial_t\tilde{A}_{ij} \sim \tilde{R}_{ij} \sim \nabla^2\tilde{\gamma}_{ij}$$

- Result: $\boxed{\text{dramatically improved stability}}$

Collapse of A Magnetized Hypermassive Star

Duez, Liu, Shapiro, Shibata & Stephens (2006a,b): axisymmetry

- Initial Seed B Field
  - Topology: purely poloidal
  - Strength: $C \equiv \max\left[\frac{B^2}{4\pi P}\right] = 2.5 \times 10^{-3}$
- B-field Amplification:
  - Winding: $\tau_A = R/v_A$
  - MRI: $\tau_{\mathrm{MRI}} \sim P_c \ll \tau_A$ (Balbus & Hawley 1991)
- Computational Challenge
  - Wavelength: $\lambda_{\mathrm{MRI}} = 2\pi v_A/\Omega \sim R/10$
  - Resolution Requirement: $\Delta \lesssim \lambda_{\mathrm{MRI}}/10$

$\Rightarrow$ To follow collapse, the evolution time must exceed $t_A \sim 75 P_c \sim 3000M$.

$\Rightarrow$ To resolve the fastest growing MRI mode, we require N^2 zones with $N \gtrsim 400$.

Central Engines For GRBs?

Duez, Liu, Shapiro, Shibata & Stephens 2006a,b*, 2006c**

- GRBS: 2 classes (BATSE, Swift, HETE, Chandra, HST)
  - Long-Soft GRBs:
    - $\tau \sim 2 - 1000$ sec;
    - in star-forming regions (spirals);
    - associated with SNs;
    - massive star collapse: 'collapsars' ?
    - Pop III collapse analogs?
  - Short-Hard GRBs:
    - $\tau \sim 10$ ms -2 sec;
    - in low star-form. regions (ellipticals);
    - SN associations excluded;
    - NS-NSs $\rightarrow$ HMNSs*? BH-NSs?

- Exciting implications for Advanced LIGO!
  - Coincidence Detections:
    - GW bursts + GRBs;
    - reasonable event rates.

- Simulations in full GRMHD:
  - required & underway!**

SMBH Spin Evolution

- Significance:
  efficiency of accretion & rate of SMBH growth depend sensitively on a/M.

- Initial Conditions: Pop III stellar collapse
  GR simulations $\Rightarrow 0 \leq a/M \lesssim 0.8$
  (Shibata & Shapiro 2002; Shibata et al. 2006)

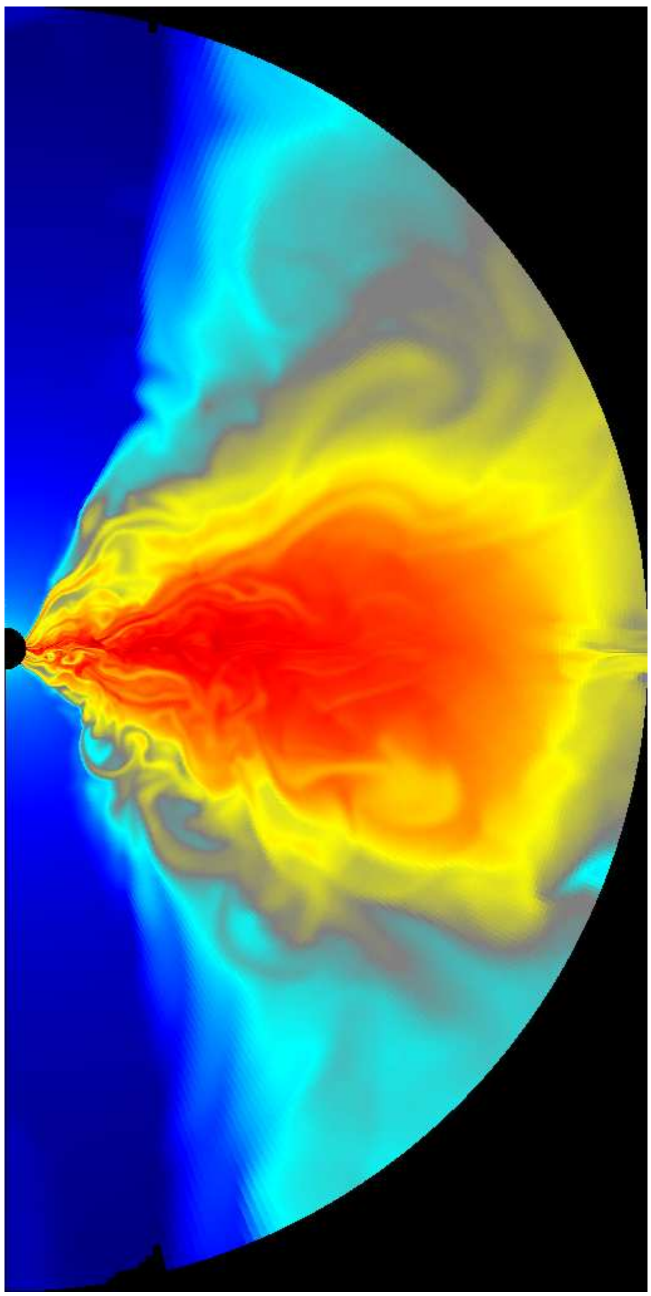
- Spin-up by major mergers
  Following binary merger, M & $a/M \approx$ values at ISCO $\Rightarrow a/M \approx 0.8 - 0.9$ for $M_1 = M_2$ (3PN & num GR calculations).
  (Damour, Cook, Baumgarte, Grandclement, ...)

- Spin-down by minor mergers
  BH merging with many smaller BHs, isotropically distributed, $\Rightarrow a/M \sim M^{-7/3}$.
  (Hughes & Blandford 2003; Gammie et al. 2004)

- Spin-equilibrium via accretion
  $a/M = 1.0$, standard thin disk (Bardeen 1970);
  $a/M = 0.998$, + photon recap. (Thorne 1974).
  $a/M \approx 0.95$, turbulent MHD disk (De Villiers et al. 2004; Gammie, Shapiro & McKinney 2004).

GRMHD Flow Snapshot for $a/M = 0.75$

McKinney & Gammie (2004); Gammie, Shapiro & Mckinney 2004



SMBH Growth By Accretion

- Efficiencies:

$$\epsilon_M \equiv L/\dot{M}_0 c^2 = \epsilon_M(a/M), \qquad \epsilon_L \equiv L/L_E,$$

$$\frac{dM}{dt} = (1-\epsilon_M)\frac{dM_0}{dt}$$

$$L_E = \frac{4\pi M \mu_e m_p c}{\sigma_T} \approx 1.3 \times 10^{46} \mu_e M_8 \text{ erg s}^{-1}.$$

- Mass and Spin Evolution:

$$\frac{dM}{dt} = \frac{\epsilon_L(1-\epsilon_M)}{\epsilon_M}\frac{M}{\tau}, \qquad \tau \equiv \frac{Mc^2}{L_E} \approx 0.45 \mu_e^{-1} \text{ Gyr}$$

$$\frac{d(a/M)}{dt} = \frac{\epsilon_L}{\epsilon_M}\frac{s}{\tau}, \quad \text{where, e.g.,}$$

$$s = \tilde{l}_{\rm ISCO} - 2\frac{a}{M}\tilde{E}_{\rm ISCO} \qquad \text{(stand. thin disk),}$$

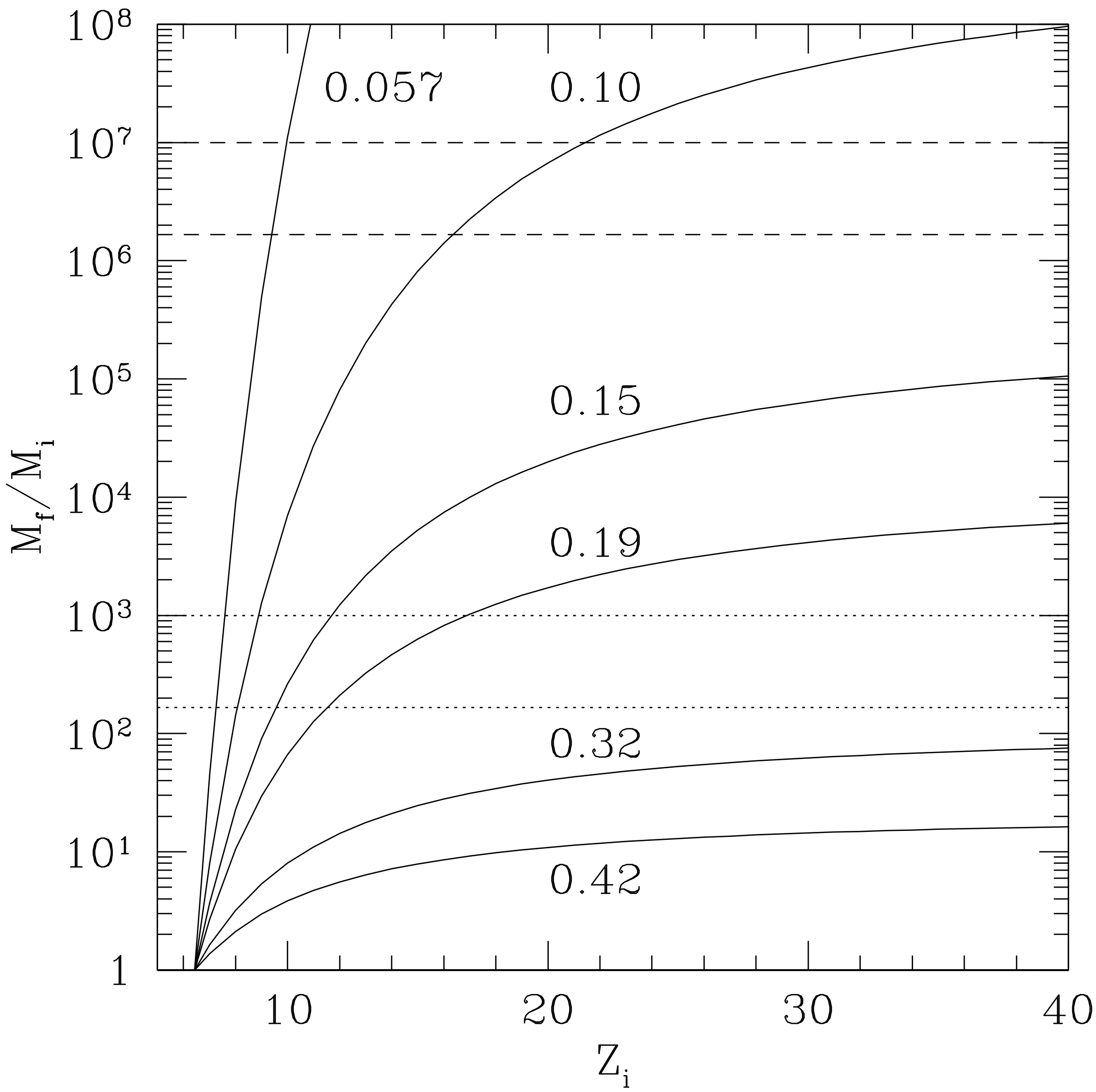
$$= 3.14 - 3.30\frac{a}{M} \qquad \text{(fit to MHD disk)}$$

- Mass Amplification at spin-equilibrium ($s = 0$):

$$M(t)/M(t_i) = \exp\left[\frac{\epsilon_L(1-\epsilon_M)}{\epsilon_M}\frac{(t-t_i)}{\tau}\right]$$

Accretion-Driven Mass Amplification

ΛCDM



$\epsilon_L = L/L_{\mathrm{E}} = 1$; curve labels: $\epsilon_M \equiv L/\dot{M}_0 c^2 = \epsilon_M(a/M)$,
$a/M = (0, 0.95, 0.998, 1) \Rightarrow \epsilon_M = (0.057, 0.19, 0.32, 0.42)$

$M_i/M_\odot$ = $100 - 600$, $M_f/M_\odot = 10^9$;
dashed = pure accretion;
dotted = 10^4 merger amplification × accretion.

Summary & Conclusions

- Key issues:
  - cosmological origin of seed SMBHs?
  - mass & spin evolution?
  - role in structure formation?

- Clues & constraints:
  - QSO 1148+5251: $z = 6.43$, $t = 0.87$ Gyr
  - $U_{QSO} \approx 0.1\ \rho_{BH} c^2$
  - $M_{BH} - \sigma_*$ correlation
  - $M_{BH} - M_{bulge}$ correlation
  - etc.

- Numerical GR:
  mature enough (at last!) to probe physics underlying cosmological formation & growth of SMBHs, e.g.,
  - collapse to seed BHs;
  - BH binary merger and recoil;
  - gravitational wave generation;
  - BH accretion;
  - etc;